University of Barcelona, Department of Fundamental Physics
iSocial Marie Curie Initial Training Networks

Individualism and collectivism in social dynamics:

contact process with stochastic opinion fluctuations in complex networks

Liudmila Rozanova
rozanova@ffn.ub.edu

January 27, 2016

Content

Contact process description

Voter model with distributed flip rate and spontaneous state changing

N is the set of contacting agents in states 0 and 1.
Agent i changes his opinion:

- to influenced by other agents. The probability of agent i copying the opinion of agent j is $P\{j|i\} = \frac{a_{ij}}{k_i} \frac{f(\lambda_j)}{\sum_{i=1}^{N} f(\lambda_i)}$,
  - k_i is the degree of agent,
  - $A = \{a_{ij}\}$ is the adjacency matrix,
  - λ_i is intrinsic activity rate of i-agent.
- spontaneously with probability ϵ_i.

System state evolution
Description of variables

Let's $n_i(t)$ is the opinion state the i-agent in the time t.

$$n_i(t+dt) = \phi_i(1-n_i(t)) + (1-\phi_i)n_i(t)(1-\xi_i) + \nu_i\xi_i \qquad (1)$$

$\phi_i(t)$, $\xi_i(t)$, and $\nu_i(t)$ are dichotomous random independent variables:

$$\phi_i(t) = \begin{cases} 1 \text{ with probability } \epsilon_i dt, \\ 0 \text{ with probability } 1-\epsilon_i dt, \end{cases} \quad \xi_i(t) = \begin{cases} 1 \text{ with probability } \lambda_i dt, \\ 0 \text{ with probability } 1-\lambda_i dt, \end{cases}$$

$$\nu_i(t) = \begin{cases} 1 \text{ with probability } \sum_{j=1}^{N} P\{j|i\}n_j(t), \\ 0 \text{ with probability } 1-\sum_{j=1}^{N} P\{j|i\}n_j(t). \end{cases}$$

System state evolution
Differential equation

Time-evolution of the average opinion:

$$\frac{\langle n_i(t+dt) - n_i(t)\rangle}{dt} = \epsilon_i - 2\epsilon_i \langle n_i(t)\rangle + \lambda_i \left[\sum_{j=1}^{N} P\{j|i\} \langle n_j(t)\rangle - \langle n_i(t)\rangle\right].$$

Ensemble average of the opinion of agent i, $\langle n_i(t)\rangle \equiv I_i$,

$$\frac{dI_i}{dt} = \lambda_i \left[\sum_{j=1}^{N} P\{j|i\} I_j - I_i\right] + \epsilon_i(1 - 2I_i). \tag{2}$$

Stochastic process $X(t)$:

$$dX(t) = M(x)dt + \sqrt{D(x)}dW, \quad (3)$$

where the drift term is

$$M(x) = \frac{\langle X(t+dt)|X(t)\rangle - \langle X(t)\rangle}{dt},$$

the diffusion term is

$$D(x) = \frac{\left\langle X^2(t+dt)|X(t)\right\rangle - \langle X(t+dt)|X(t)\rangle^2}{dt},$$

dW is the differential Wiener process.

Let's $l_k(t) = \frac{1}{N_k}\sum_{i\in k} n_i(t)$ and $\lambda_k \equiv \lambda$, $\epsilon_k \equiv \epsilon$ are fixed coefficients for all $i \in k$.

Langevin equation for general case:

$$\frac{dl_k(t)}{dt} = \epsilon(1-2l_k(t)) - \lambda l_k(t) + \lambda\sum_j P\{j|i\}l_j(t)$$
$$+\tau_k(t)\sqrt{\frac{\epsilon}{N_k}+\frac{\lambda}{N_k}\left[(1-2l_k(t))\sum_j P\{j|i\}l_j(t)+l_k(t)\right]}, \qquad (4)$$

where $\tau_k(t)$ is Gaussian white noises.

Let's $\rho(t)$ is the density of nodes in 1-state, then

$$\frac{d\rho(t)}{dt} = \epsilon - 2\epsilon\rho(t) + \tau(t)\sqrt{\frac{1}{N}\left[\epsilon + 2\lambda\rho(t)(1-\rho(t))\right]}. \qquad (5)$$

For describing fluctuation dynamics we investigate the effective potential

$$V_{eff}(\rho) = \ln\frac{1}{N}\left[\epsilon + 2\lambda\rho(1-\rho)\right] - 2\int\frac{N\epsilon(1-2\rho)}{\epsilon + 2\lambda\rho(1-\rho)}d\rho, \qquad (6)$$

V_{eff} has an extremum when $\rho = 1/2$, it is a maximum if $\lambda/N > \epsilon$ and minimum otherwise.

Split network: fast-slow agents

Model description

Two groups - fast and slow agents,

- N_f, N_s - number of agents in each group,
- λ_f and λ_s - rate parameters,
- $k_{fs} = \frac{f(\lambda_f)N_f}{f(\lambda_s)N_s}$,
- $\rho_f(t), \rho_s(t)$ - density of fast and slow agents in state 1,
- ϵ - spontaneous flip probability.

System of differential equations:

$$\frac{d\rho_f(t)}{dt} = \epsilon(1-2\rho_f) + \frac{\lambda_f}{1+k_{fs}}(\rho_s - \rho_f) + \sqrt{\frac{1}{N_f}\left[\epsilon + \lambda_f \frac{\rho_s + \rho_f(1+2k_{fs} - 2\rho_s - 2k_{fs}\rho_f)}{1+k_{fs}}\right]}\tau_f(t),$$

$$\frac{d\rho_s(t)}{dt} = \epsilon(1-2\rho_s) + \frac{\lambda_s k_{fs}}{1+k_{fs}}(\rho_f - \rho_s) + \sqrt{\frac{1}{N_s}\left[\epsilon + \lambda_s \frac{\rho_f k_{fs} + \rho_s(k_{fs} + 2 - 2\rho_f k_{fs} - 2\rho_s)}{1+k_{fs}}\right]}\tau_s(t).$$

If ϵ has the same order as λ_s, $\Rightarrow$ fix $\rho_s \Rightarrow$ consider just one Langevin equation.
The effective potential has a single extremum at approximately

$$\rho_s \approx \rho_f \approx \frac{1}{2}.$$

It is a maximum if

$$N_f(2\epsilon + \lambda_f) + 2k_{fs}(\epsilon N_f - \lambda_f) < 0, \tag{7}$$

and minimum in another case. Transition from minimum to maximum can only happen if $\lambda_f > \epsilon N_f$.

Split network: fast-slow agents

Conditions of the effective potential maximum

$$x \equiv \frac{2f(\lambda_f)}{N_s f(\lambda_s)}, y \equiv \frac{\lambda_f}{\epsilon N_f}.$$

The effective potential maximum possible only if

$$x > 1 \tag{8}$$

and

$$y > \frac{x + \frac{2}{N_f}}{x - 1}. \tag{9}$$

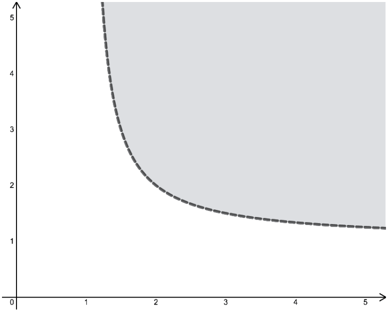
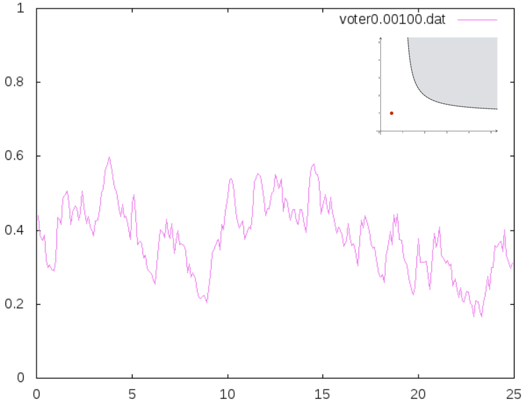


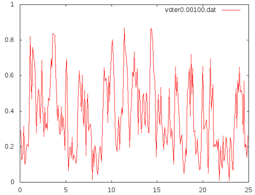
Figure 1: Effective potential condition in XY space, $N_f = 1000$.

Split network: fast-slow agents

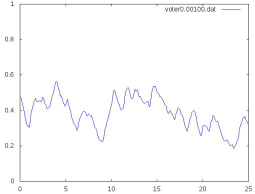
Simulation: evolution of the fraction of agents in the 1-state



(a) Fast+slow agents



(b) Fast agents

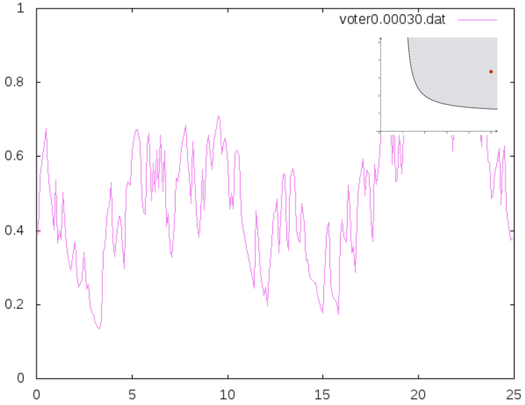


(c) Slow agents

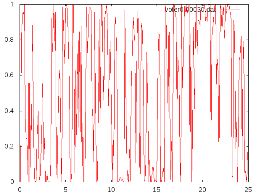
Figure 2: $N_f = 1000$ fast, $N_s = 4000$ slow agents, $\lambda_f = 10^3 \lambda_s, \epsilon = 0.001$.

Split network: fast-slow agents

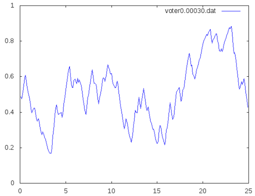
Simulation: evolution of the fraction of agents in the 1-state



(a) Fast+slow agents



(b) Fast agents

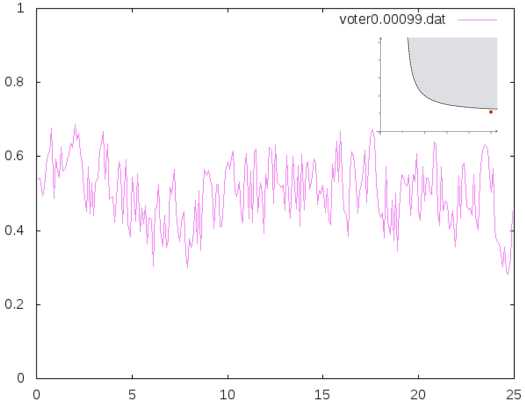


(c) Slow agents

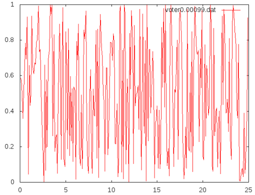
Figure 3: $N_f = 1000$ fast, $N_s = 4000$ slow agents, $\lambda_f = 10^4 \lambda_s, \epsilon = 0.0003$.

Split network: fast-slow agents

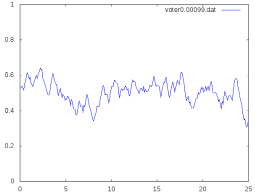
Simulation: evolution of the fraction of agents in the 1-state



(a) Fast+slow agents



(b) Fast agents

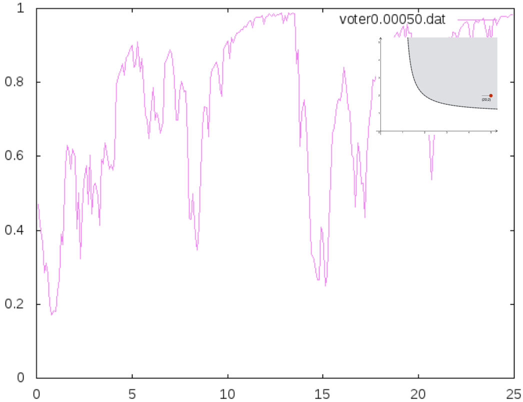


(c) Slow agents

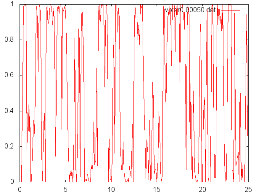
Figure 4: $N_f = 1000$ fast, $N_s = 4000$ slow agents, $\lambda_f = 10^4 \lambda_s, \epsilon = 0.00099$.

Split network: fast-slow agents

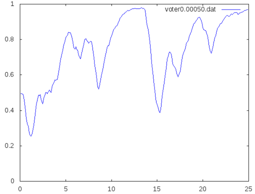
Simulation: evolution of the fraction of agents in the 1-state



(a) Fast+slow agents



(b) Fast agents

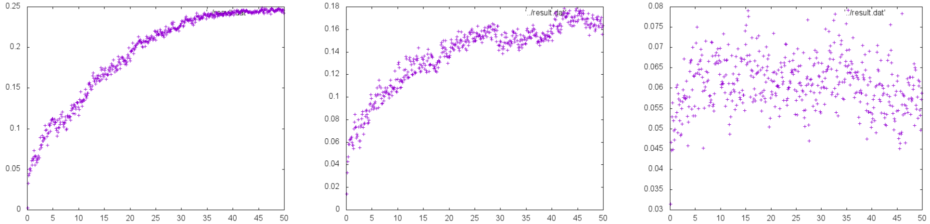


(c) Slow agents

Figure 5: $N_f = 1000$ fast, $N_s = 1000$ slow agents, $\lambda_f = 10^4 \lambda_s, \epsilon = 0.0005$.

Split network: fast-slow agents
Simulation: criterion of the effective potential maximum

Variance curve $\langle(\rho_f - 1/2)^2(t)\rangle$ slope indicates the existence of the effective potential maximum.



(a) $\epsilon = 0$; (b) $\epsilon = 0.0002$; (c) $\epsilon = 0.001$.

Figure 6: Variances of the fraction of fast agents in 1-state, $N_f = 1000$, $N_s = 4000$, $\lambda_f = 10^4\lambda_s$.

Thank you for your attention!